Configuration Mixing in Relativistic Quark Models

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Abstract

Starting from the standard and commonly used relativistic nucleon wave function, we show that residual interactions based on gluon or Goldstone boson exchanges generate additional components leading to configuration mixing. This result suggests that realistic nucleon wave functions can be expected to be far more complex than seemingly successful form factor fits in relativistic quark models would have one believe.

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I. INTRODUCTION

The nonrelativistic quark model (NQM) explains qualitatively many of the strong, electromagnetic and weak interaction properties of the nucleon and other octet and decuplet baryons in terms of three valence quarks whose dynamics is motivated by quantum chromodynamics (QCD), the gauge field theory of the strong interaction. Lacking presently

a complete solution of QCD at low and intermediate energies, where nonperturbative effects are essential, quark models are useful as they provide physical insight and can easily be related to empirical analyses. However nonrelativistic approaches face difficulties, when quantitative results are required, e.g., at larger momentum transfers, or even at $q^2 = 0$ for some weak interaction observables[1].

A frame for a systematic approach to a relativistic few-body theory is provided by light front dynamics [2]. This is intuitively appealing since it uses a Hamiltonian concept and is thus easy to interprete. As in the nonrelativistic case the phenomenon of confinement is modeled by a confining force that can be chosen to be three dimensional. Residual interactions between quarks related to "hyperfine" splittings may be calculated in light cone time ordered perturbation theory. By now several residual interactions have been used in the context of nonrelativistic models, gluon type, Goldstone boson type and instanton type [3] interactions that lead to characteristic spin, isospin, and orbital mass splittings in the baryonic spectrum [4–6]. All these interactions induce configuration mixing that beside mass splitting shows qualitatively new effects. None of these interactions have been considered in the light front framework so far and merely "ground state" wave functions of the nucleon have been used. Since a complete solution of the relativistic three-quark problem on the light front is still missing, we provide a further step towards this aim and incorporate the Goldstone boson and the gluon exchange: We calculate the spin-isospin matrix elements that appear in the three body equation. We show that both interactions lead to configuration mixing.

Due to the spontaneous chiral symmetry breakdown (χSB) of QCD, the effective degrees of freedom at the scale Λ_{QCD} are quarks along with Goldstone bosons. Chiral field theory involves the effective strong interaction commonly used in chiral perturbation the-

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[1] One of the early and obvious examples for a relativistic effect is the parity violating weak current that leads to the axial coupling of the nucleon to the W boson, g_A [1].

ory (χPT [7]) and applies at scales from Λ_{QCD} up to the chiral symmetry restoration scale $\sim \Lambda_\chi = 4\pi f_\pi \sim 1.17$ GeV, where $f_\pi = 0.093$ GeV is the pion decay constant. It provides a scenario inside hadrons at distances that are smaller than the confinement scale Λ_{QCD}^{-1}, but larger than distances where perturbative QCD applies, in which the effective degrees of freedom of the strong interaction are dynamical quarks (with a mass depending on their momentum) along with the SU(3) octet of Goldstone bosons comprising the pions, kaons and the eta meson. Chiral field theory dissolves a dynamical quark into a current quark and a cloud of virtual Goldstone bosons.

II. RELATIVISTIC NUCLEON WAVE FUNCTION

Within the nonrelativistic quark model (NQM) baryons are described as Fock states of three quarks - in the simplest case. Although not derived from first principles of QCD, the bulk properties of the nucleon can be described in a qualitative fashion. Meanwhile experience in quark modeling and various experiments at high energies established the necessity of using a relativistic description of the nucleon. The difficulties of a proper relativistic description of bound states are also obvious and, to date, neither a general solution exists nor one that would satisfy all required conditions. Among the promising candidates for a covariant description of a few-body bound state is one utilizing the light front form. In the context of the two-body problem the connection between the four dimensional covariant form of the Bethe-Salpeter amplitude and the three-dimensional light front wave function has been described explicitly and their differences have been discussed in Ref. [8].

So far the three quark bound state problem on the light front for the nucleon has been tackled utilizing several (sometimes drastic) approximations, e.g. spinless quarks within a Weinberg-type or a Faddeev-type equation [9,10]. To evaluate currents with appropriate boost properties the wave functions can be constructed by rewriting the nonrelativistic results in light front variables using Melosh rotated Pauli spinors [11].

A particular problem in the three quark system is the proper modeling of the confining

force that presently seems to require a three dimensional form.

Lacking at present a complete solution our strategy is as follows. For the time being we assume that the three quark wave function can be expanded into a set of square integrable functions. As an example harmonic oscillator functions have widely been used in the past. Indeed one may assume a three dimensional harmonic oscillator confining potential U that will be added to the mass (squared) operator, M_0^2, of the free three quark system following the Bakamjian-Thomas construction. This leads to harmonic oscillator functions $\{R_n\}$ for the configuration/momentum space. For the spinless case this approach has been shown to lead to the proper Regge trajectories, both for mesons and baryons, provided some conditions on the parameters are fulfilled [9].

The proper coordinates to describe the evolution of a particle in light cone dynamics are the light cone time $\tau = x^0 + x^3$ and the coordinates $(x^-, \boldsymbol{x}_\perp) = (x^0 - x^3, (x_1, x_2))$ arranged as $x = (\tau, x^-, \boldsymbol{x}_\perp)$. The momenta of the quark in the nucleon moving frame are denoted by p_i and

$$p^\mu = \left(p^+ = p^0 + p^3, p^- = \frac{m^2 + \boldsymbol{p}_\perp^2}{p^+}, \boldsymbol{p}_\perp \right). \tag{1}$$

In the light-cone approach all particles are on their mass shell, i.e. $m^2 = p^+p^- - \boldsymbol{p}_\perp^2$ and the spectrum of the total momentum, P^+, is bounded from below. For all particles $p_i^+ \geq 0$, and the kinematically invariant light cone fractions are $x_i = p_i^+/P^+$. Since light cone quarks are off the p^- shell the three-body proper coordinates are given in terms of the $+$ and $\perp$ components only,

$$\begin{aligned} P &= \sum_i p_i, \\ q_3 &= \frac{x_2 p_1 - x_1 p_2}{x_1 + x_2}, \\ Q_3 &= (1 - x_3) p_3 - x_3 (p_1 + p_2), \end{aligned} \tag{2}$$

where the first equation is the total momentum P and the other two are coordinates for the pair q_3 and for the odd quark Q_3 (given in the (12)3 coupling scheme, viz. channel 3. Other channels are given by cyclic permutation of 1,2,3.). Note that $q_3^+ = Q_3^+ = 0$.

Several boosts are kinematic, whereas rotations ($\boldsymbol{L}_\perp$) are not. The kinematic light cone generators are transitive on the null plane, so that the wave function is known everywhere, once it is known in the rest frame of a bound few-body system. Finally, the total momentum separates rigorously from the internal motion in momentum space, which is not the case in the instant form, where all boosts are interaction dependent and rotations are kinematic.

By now the structure of the ground state (i.e. without additional momentum excitations and with symmetric momentum space wave function) in the light front formalism is well understood and tested. In the rest frame of the three particle system with free total momentum, $\overset{\circ}{P}= (M_0, M_0, \mathbf{0}_\perp)$, the nucleon ground state may be written in the following way:

$$\Psi_N(123) = \psi_N(123)\; R_0(\boldsymbol{k}_{i,\perp}, k_i^+), \tag{3}$$

where $1 \equiv \{k_1, \lambda_1, \tau_1, \dots\}$ denotes the momentum k_1, helicity λ_1, isospin τ_1 of particle 1 and ψ_N is the relativistic wave function depending on spin and isospin. The Dirac structures necessary to specify the spin-isospin components are given in Table I. However, because of the symmetry restrictions (S_3 and parity of the total wave function) the number of independent ground state components reduces to three linearly independent symmetric spin-flavor states (see e.g. [12] and refs. therein), viz. [2]

$$\psi_N(123) = \sum_{\kappa=0}^{2} c_\kappa I_\kappa(123)\; u_1\, u_2\, u_3. \tag{4}$$

The quark spinors are denoted by $u_i = u_{LC}(p_i, \lambda_i)\chi_i$ where the light cone spinors $u_{LC}(p_i, \lambda_i)$ are given explicitly, e.g., in Ref. [13] and χ_i are the isospinors. The spin-isospin wave functions invariants are given by

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[2]Different momentum wave functions, $R_0^{(\kappa)}$, for various spin-flavor invariants I_κ are not ruled out and would, in fact, be needed to describe different momentum dependence of the proton's charge and magnetic form factors according to recent measurements at the Thomas Jefferson Research Laboratory (JLab) [14]

$$I_0(123) = \bar{u}_1\gamma_5 G\bar{u}_2^T\ \bar{u}_3 u_\lambda + (23)1 + (31)2, \tag{5}$$

$$I_1(123) = \bar{u}_1\gamma^\mu \boldsymbol{\tau} G\bar{u}_2^T\ \bar{u}_3\gamma_5\gamma_\mu \boldsymbol{\tau} u_\lambda + (23)1 + (31)2, \tag{6}$$

$$I_2(123) = \bar{u}_1\gamma\cdot P\gamma_5 G\bar{u}_2^T\ \bar{u}_3 u_\lambda + (23)1 + (31)2, \tag{7}$$

where $G = i\tau_2 C$, and $C = i\gamma_0\gamma_2$ is the charge conjugation matrix and T denotes transposition. The coefficients c_κ have to be determined from a dynamical equation. The larger number of nucleonic components even in the ground state (compared to only one in the nonrelativistic case) is related to negative energy contributions in the wave function. This is similar to the deuteron as a bound two nucleon system and gives rise to P states that are of pure relativistic origin. In the past, the coefficients c_κ have been chosen in a way that the resulting ground state relativistic wave function is restricted to contain only positive energy components. As a consequence only one independent invariant spin-flavor structure arises. This can best been seen in the Bargmann Wigner basis [15], but is also clear from the nonrelativistic wave function that is Melosh boosted to the light cone. In its turn this particular relativistic wave function can be said to be a straightforward generalization (based on Melosh rotations) of the nonrelativistic three quark s-wave function (that contains positive energy Fock components only). This particular linear combination, $G_2 + G_6$, is then given by

$$\psi_0(123) = \mathcal{N}\sum_{\lambda_i}\Big[\Big(\bar{u}_1(\gamma\cdot P + M_0)\gamma_5 G\bar{u}_2^T\Big)\left(\bar{u}_3 u_N(P)\right) + (23)1 + (31)2\Big]\, u_1 u_2 u_3, \tag{8}$$

where $\mathcal{N}$ denotes the normalization and the nucleon spinor $u_N(P)$ depends on total P. In other words, when the mixed antisymmetric nucleon spin-isospin state is Melosh boosted, the combination $G_2 + G_6$+permutations of Table I and Eq. 8 is obtained. When the mixed symmetric state is similarly treated another combination, $G_3 - G_5 - G_8$, is obtained. Of course, in the uds basis both lead to the same totally symmetric spin-isospin nucleon wave function. The excited states may be constructed in a similar fashion, as has been shown in Ref. [15].

The equation of motion for the three quark bound state Ψ_N may be written [9,16,17] according to the Bakamjian-Thomas prescription (BT) as

$$\Psi_N = G_3^{(0)}(m_N^2)(V^{(1)} + V^{(2)} + V^{(3)})\Psi_N, \tag{9}$$

where we have assumed a two-body interaction V and utilized the three-body channel notation (i.e. $V^{(3)} = V(12)$ etc.). The free Green function $G_3^{(0)}$ is given by

$$\begin{aligned} G_3^{(0)-1}(m_N^2) &= P^+ \left(P^- - \sum_{j=1}^{3} p_j^- \right) \\ &= m_N^2 + \frac{1 - x_3}{x_1 x_2}\, q_3^2 + \frac{1}{x_3(1 - x_3)}\, Q_3^2 - \sum_{j=1}^{3} \frac{m_j^2}{x_j} \end{aligned} \tag{10}$$

that has to be evaluated at the eigenvalue m_N of Ψ_N, and q_3 and Q_3 are four-vectors. Note that $G_3^{(0)}$ is independent of the specific channel chosen for the three body coordinates (here channel 3). The *uds* basis used in Eqs. 7 and 8 suggests a Faddeev decomposition of the wave function, $\Psi_N = \Psi_N^{(1)} + \Psi_N^{(2)} + \Psi^{(3)}$ shown in Fig. 1. In this notation the Faddeev components of the three-body bound state are given by ($\alpha = 1, 2, 3$)

$$\Psi_N^{(\alpha)} = G_3^{(0)}(m_N^2)\ V^{(\alpha)}\, \Psi_N, \qquad \alpha = 1, 2, 3. \tag{11}$$

A diagrammatical form of this equation is shown in Fig. 2. The integrals appearing in Eqs. 9 and 11 are written more explicitely (e.g. in channel 3)

$$\begin{aligned} (V^{(3)}\Psi_N)(1'2'3') &= \sum_{123} V^{(3)}(1'2', 12)\ \delta_{3'3}\Psi_N(123) \\ &= \int \frac{d^2\boldsymbol{q}_{3\perp} dx_1}{2(2\pi)^3 x_1}\ V^{(3)}(x_1', \boldsymbol{q}_{3\perp}'; x_1, \boldsymbol{q}_{3\perp})\ \Psi_N(123'), \end{aligned} \tag{12}$$

The last equation is written for $\boldsymbol{P}_\perp = 0$, and spin isospin indices suppressed. Note that $x_1 + x_2 + x_3' = 1$ must be fulfilled.

This integral will now be evaluated in the following way: We introduce the nucleon wave function given in Eq. 3 and calculate the corresponding diagrams of Figs. 3-5 using light front perturbation theory.

The diagram of Fig. 3a, e.g., represents the π^0 exchange between quark 1 and 2 with momentum $k = p_1 - p_1'$ for the $\perp, +$ components. In this case the integral, Eq. 12 becomes (up to a Jacobian and constants),

$$\int \frac{dk^+ d^2\boldsymbol{k}_\perp}{2(2\pi)^3 k^+} \frac{v^{(3)}(k^+,\boldsymbol{k}_\perp)}{E_{12}} R_0(k^+,\boldsymbol{k}_\perp,3')\,\psi_N(123'). \tag{13}$$

The light front energy denominator E_{12} for this particular case is given by

$$E_{12} = m_N^2 - E_3' - E_\pi - E_1' - E_2, \tag{14}$$

with on-shell light-cone energies E_i and E_π, while the remaining vertex function denoted here by $v^{(3)} = v(12)$ will be defined in the next section.

In the remaining part of the paper we investigate this integral and evaluate the spin isospin terms. Presently we focus on two types of residual interactions between the quarks, Goldstone boson exchange and gluon exchange. We will show that both interactions lead to configuration mixing in the ground state (nucleon) wave function that has not yet been considered for relativistic quark models.

We note here that for the two nucleon case integrals of this type have been evaluated for the Bethe-Salpeter approach [8] and the light front approach [18]. The respective equations can be solved by iterations. Starting from a nonrelativistic wave function already the first order approximation generates all the relativistic components and has been quite useful to study qualitative features of the wave functions. The additional wave function components arising are indeed essential for the description of deuteron break up even at threshold energies as they are related to the standard relativistic pair current corrections in a nonrelativistic frame work, see [8]. In the case of magnetic moments of the deuteron these additional wave function components have been shown to quantitatively agree with the standard expansion in terms of pair currents [19].

III. GOLDSTONE BOSON AND GLUON EXCHANGE

We discuss two different types of residual interactions. One based on gluon exchange (v_G) has been thoroughly studied in nonrelativistic models. Another based on Goldstone boson exchange (v_5) has recently been suggested as a possibility to incorporate the concept of chiral symmetry breaking into quark modeling [6].

If the chiral symmetry breakdown is based on $SU(3)_L \times SU(3)_R$, then the effective interaction between quarks and the octet of Goldstone boson (GB) fields Φ_i involves the axial vector coupling

$$\mathcal{L}_{int} = -\frac{g_A}{2f_\pi} \sum_{i=1}^{8} \bar{q}\partial_\mu \gamma^\mu \gamma_5 \lambda_i \Phi_i q \tag{15}$$

that is well known from soft-pion physics. In Eq. 15, the λ_i, $(i = 1, 2, ..., 8)$ are Gell-Mann's SU(3) flavor matrices, and g_A is the dimensionless axial vector-quark coupling constant that is taken to be one here. As a consequence, the polarization of quarks flips in chiral fluctuations, $q_{\uparrow,\downarrow,} \to q_{\downarrow,\uparrow} + GB$, into pseudoscalar mesons of the SU(3) flavor octet of Goldstone bosons, but for massive quarks the non-spinflip transitions from $\gamma_\pm \gamma_5 k^\pm$ that depend on the quark masses are not negligible. Let us also emphasize that, despite the nonperturbative nature of the chiral symmetry breakdown, the interaction between quarks and Goldstone bosons is small enough for a *perturbative expansion* to apply.

The nucleon wave function based on the Pauli-Melosh basis contains no small Dirac spinor components. Hence its spin function is uniquely specified by the S-wave 3-quark wave function in the case of no configuration mixing. In terms of the Dirac-Melosh basis its spin wave function involves the combination $M_0 G_2 + G_6$ [15] of spin invariants. Many electroweak form factor calculations are based on this wave function. We wish to show first that effective interactions like pion exchange or gluon exchange generate many other relativistic spin invariants starting from this component alone.

To be specific we will drop the momentum wave function of the proton and evaluate the term $v_5(12)\psi_N$ that is the spin-isospin part of the ground state (nucleon) wave function. That is, we consider a 1-2 symmetric neutral pion exchange interaction in channel 3 of the nucleon equation of motion (see Eq. 11, Figs. 2 and 3, π^0 exch. with uud flavor part), i.e.

$$\begin{aligned} v_5(12)\psi_N &= v_5(12)(\psi_N^{(1)} + \psi_N^{(2)} + \psi_N^{(3)}) \\ &= 4m_q^2\ \bar{u}_1'\gamma_5 u_1(\bar{u}_2'\gamma_5 u_2)\Big(\bar{u}_3[P]\gamma_5 C\bar{u}_1^T(\bar{u}_2 u_N) + \bar{u}_3[P]\gamma_5 C\bar{u}_2^T(\bar{u}_1 u_N)\Big) \\ &= -\bar{u}_3'[P]\gamma\cdot\bar{k}C\bar{u}_1'^T(\bar{u}_2'\gamma_5\gamma\cdot k u_N) - \bar{u}_3'[P]\gamma\cdot\bar{k}C\bar{u}_2'^T(\bar{u}_1'\gamma_5\gamma\cdot\bar{k}u_N). \end{aligned} \tag{16}$$

The positive-energy Dirac projection operator $[P] \equiv (\gamma \cdot P + M_0)$ in Eq. 16 originates from the Melosh boost of the quarks to the light cone as explained in the previous section. Here, the isospin part is treated separately and therefore the light cone quark spinors are given by $u_i = u(p_i, \lambda_i)$, $u'_i = u(p'_i, \lambda'_i)$. Note that the second term in Eq. 16 is related to the first by $1 \leftrightarrow 2$ symmetry.

The instantaneous quark propagator has been included in the quark propagator (i.e. spin sum) by replacing the quark minus momentum component p_i^- by the modified value [20] $\tilde{p}_i^- = P^- - \bar{p}_j^- - \bar{p}_k^- - \bar{k}^-$, where the bar denotes the on-shell quantity and k the pion momentum. For $v_5(12)$, $\tilde{p}_2^-$ for example, may be written as

$$\begin{aligned} \tilde{p}_2^- &= (P^- - \sum_i \bar{p}_i^-) + \bar{p}_2^- - \bar{k}^- \\ &= \Delta P + (\bar{p}_2^- - \bar{p}_2'^- - 2\bar{k}^-) + \bar{p}_2'^- + \bar{k}^- \equiv \Delta p_2 + \bar{p}_2'^- + \bar{k}^-, \end{aligned} \tag{17}$$

$$\Delta P = P^- - \sum_{i=1}^{3} \bar{p}_i^- . \tag{18}$$

Hence, when we sum over intermediate quark helicities for the first term in Eq. 16 using the kinematics $p'_1 = p_1 - k$, $p'_2 = p_2 + k$ for the $+, \perp$ components in $\sum_{\lambda_2} u_2 \bar{u}_2 = (\gamma \cdot p_2 + m_q)/2m_q$ in conjunction with the free Dirac equation for u'_2, we obtain

$$\begin{aligned} 2m_q \sum_{\lambda_2} \bar{u}'_2 \gamma_5 u_2 \; \bar{u}_2 u_N &= \bar{u}'_2 \gamma_5 (\gamma \cdot \tilde{p}_2 + m_q) u_N \\ &= \bar{u}'_2 \gamma_5 \gamma \cdot \bar{k} u_N + \Delta p_2 (\bar{u}'_2 \gamma_5 \gamma^+ u_N). \end{aligned} \tag{19}$$

The small off-minus-shell second term in Eq. 19 (and elsewhere) will be neglected in the following for simplicity and merely indicated by $\cdots$, but will be considered and numerically investigated elsewhere. Note that the pion momentum k is the integration variable in the nucleon equation of motion.

In order to generate nucleon spin invariants in Table I, we Fierz transform each term on the right of Eq. 16 using the scalar line of Table II. This produces five terms which collapse to the following two upon using their $(1 \leftrightarrow 2)$ symmetry or antisymmetry,

$$\begin{aligned} v_5(12)\psi_N = &-\frac{1}{2}\; \bar{u}'_1 \gamma_\mu C \bar{u}_2'^T \; (\bar{u}'_3 [P] \gamma_5 \gamma \cdot k \gamma^\mu \gamma \cdot k u_N) \\ &+\frac{1}{4}\; \bar{u}'_1 \sigma_{\mu\nu} C \bar{u}_2'^T \; (\bar{u}'_3 [P] \gamma_5 \gamma \cdot k \sigma^{\mu\nu} \gamma \cdot k u_N), \end{aligned} \tag{20}$$

whose 1-2 symmetric first matrix elements already have the canonical form of the nucleon spin invariants of Table I. The second part is reorganized to yield

$$
\begin{aligned}
v_5(12)\psi_N = &-\bar{u}_1'\gamma_\mu C\bar{u}_2'^T\ \bar{u}_3'\gamma_5\Big[-M_0k^2\gamma^\mu + k^2P^\mu + 2M_0k^\mu\gamma\cdot k - 2k\cdot Pk^\mu\Big]u_N \\
&+\frac{1}{2}\bar{u}_1'\sigma_{\mu\nu}C\bar{u}_2'^T\ \bar{u}_3'\gamma_5\Big[ik^2(\gamma^\mu P^\nu - \gamma^\nu P^\mu) + 2i(P^\mu k^\nu - P^\nu k^\mu)\gamma\cdot k \\
&+2ik\cdot P(k^\mu\gamma^\nu - k^\nu\gamma^\mu)\Big]u_N
\end{aligned}
\tag{21}
$$

modulo small off-minus-shell terms. The first term, $-M_0k^2$, in the brackets gives a G_3 contribution in Eq. 21, the second and fifth give G_5 and G_8, respectively. Altogether we obtain from Eq. 21

$$
\begin{aligned}
v_5(12)\psi_N = &-k^2M_0\bar{u}_1'\gamma_\mu C\bar{u}_2'^T\ \bar{u}_3'\gamma^\mu\gamma_5u_N - k^2\bar{u}_1'\gamma\cdot PC\bar{u}_2'^T\ \bar{u}_3'\gamma_5u_N \\
&-k^2\bar{u}_1'i\sigma_{\mu\nu}P^\nu C\bar{u}_2'^T\ \bar{u}_3'\gamma^\mu\gamma_5u_N + 2k\cdot P\bar{u}_1'\gamma\cdot kC\bar{u}_2'^T\ \bar{u}_3'\gamma_5u_N \\
&-2k\cdot P\bar{u}_1'i\sigma_{\mu\nu}k^\nu C\bar{u}_2'^T\ \bar{u}_3'\gamma_5\gamma^\mu u_N - 2M_0\bar{u}_1'\gamma\cdot kC\bar{u}_2'^T\ \bar{u}_3'\gamma_5\gamma\cdot ku_N \\
&+2\bar{u}_1'i\sigma_{\mu\nu}P^\mu k^\nu C\bar{u}_2'^T\ \bar{u}_3'\gamma_5\gamma\cdot ku_N.
\end{aligned}
\tag{22}
$$

In Eq. 22 the first three terms can be combined to the single *proper* vector-spin invariant of the nucleon with projector $[P]$ from the Melosh boost for each quark, viz.

$$
-\frac{k^2}{4M_0^2}\ \bar{u}_1'[P]\gamma_\mu C(\bar{u}_2'[P])^T\ \bar{u}_3'[P]\gamma^\mu\gamma_5u_N, \tag{23}
$$

which has only positive energy components. The last four terms containing k dependent spin invariants will be addressed in the next section, where it is shown that they give rise to N and N*-type spin invariants through the angular $\boldsymbol{k}_\perp$ integration. Symbolically we may write Eq. 22 as

$$
v_5(12)(G_2 + G_6) \sim G_3 - G_5 - G_8 + \cdots \tag{24}
$$

in terms of the spin invariants of Table I. This says that a pseudoscalar interaction $v_5(12)$ between quarks 1 and 2 acting on the nucleon spin invariants G_2 and G_6 is off-diagonal and makes transitions to the nucleon spin invariants G_3, G_5 and G_8 so far, but more will follow.

We also notice from missing $[P]$'s next to the quark spinors in the k dependent terms that small Dirac components are generated by these interactions, so that the *Pauli-Melosh basis is clearly too limited.*

The isospin matrix element corresponding to the neutral pion exchange $v_5(12)$ is straightforward to evaluate

$$(\chi_3^T \tau_3 i\tau_2 \chi_1)(\chi_2^T \tau_3 \chi_\uparrow) = 1,\ \chi_1 = |u\rangle,\ \chi_2 = |u\rangle,\ \chi_3 = |d\rangle. \tag{25}$$

The same result is valid for $1 \leftrightarrow 2$. The isospin matrix element for the charged pion exchange $v_5^+(13)$ that we consider next is evaluated similarly, viz.

$$-\chi_3^T \tau^- i\tau_2 (\tau^+)^T \chi_1 (\chi_2^T \chi_\uparrow) = -2, \tag{26}$$

where $\tau^\pm = \mp\frac{1}{\sqrt{2}}(\tau_1 \pm i\tau_2),\ \vec{\tau}\cdot\vec{\tau} = \tau_3\tau_3 - (\tau^+\tau^- + \tau^-\tau^+)$.

To demonstrate the case for the π^+ exchange interaction we consider channels 1 and 2, since in the uds basis and for the proton quarks 1 and 2 are both up quarks and no charged exchange is possible between them. We consider the 1-2 symmetric combination $\mathcal{G}_0[v_5^+(23) + v_5^+(32) + v_5^+(13) + v_5^+(31)]$ on G_2, where the Green function symbol $\mathcal{G}_0$ denotes various energy denominators. To preserve the uud flavor order in the nucleon wave function we need to relabel the u and d quarks $2 \leftrightarrow 3$ and $1 \leftrightarrow 3$, respectively. Thus we start from

$$\begin{aligned}
&\left[(E_{23}^{-1} + E_{32}^{-1})v_5^+(23) + (E_{13}^{-1} + E_{31}^{-1})v_5^+(13)\right]\psi_N \\
&= 4m_q^2(E_{23}^{-1} + E_{32}^{-1})\ \bar{u}_2'\gamma_5 u_2\ \bar{u}_3'\gamma_5 u_3 \left(\bar{u}_3[P]\gamma_5 C\bar{u}_1^T\ \bar{u}_2 u_N + \bar{u}_3[P]\gamma_5 C\bar{u}_2^T\ \bar{u}_1 u_N\right) \\
&\quad +4m_q^2(E_{13}^{-1} + E_{31}^{-1})\ \bar{u}_1'\gamma_5 u_1\ \bar{u}_3'\gamma_5 u_3 \left(\bar{u}_3[P]\gamma_5 C\bar{u}_1^T\ \bar{u}_2 u_N + \bar{u}_3[P]\gamma_5 C\bar{u}_2^T\ \bar{u}_1 u_N\right) \\
&= -(E_{23}^{-1} + E_{32}^{-1})\ \Big[\bar{u}_3'\gamma_5\gamma\cdot k[P]C\bar{u}_1'^T\ \bar{u}_2'\gamma\cdot k\gamma_5 u_N \\
&\qquad\qquad +\bar{u}_3'\gamma_5\gamma\cdot k[P]\gamma\cdot kC\bar{u}_2'^T\ \bar{u}_1' u_N\Big]|udu\rangle + (1\leftrightarrow 2)|duu\rangle \\
&= -(E_{23}^{-1} + E_{32}^{-1})\ \Big[\bar{u}_2'\gamma_5\gamma\cdot k[P]\gamma_5 C\bar{u}_1'^T\ \bar{u}_3'\gamma\cdot k\gamma_5 u_N \\
&\qquad\qquad +\bar{u}_2'\gamma_5\gamma\cdot k[P]\gamma\cdot kC\bar{u}_3'^T\ \bar{u}_1' u_N\Big]|uud\rangle + (1\leftrightarrow 2)\ |uud\rangle
\end{aligned} \tag{27}$$

requiring no Fierz rearrangement for the (12)3 and (21)3 terms. The flavor dependence is

displayed on the right hand side of Eq. 27 to avoid confusion in a comparison with Eq. 16. Equation 27 will be continued in the next Section after the angular $\boldsymbol{k}_\perp$ integration.

IV. RELATIVISTIC COUPLED EQUATIONS

We now turn to evaluate the integral given in Eq. 13 in first order approximation. The angular $\boldsymbol{k}_\perp$ integration is performed using the identity

$$\int d^2\boldsymbol{k}_\perp \boldsymbol{a}_\perp \cdot \boldsymbol{k}_\perp \ \boldsymbol{b}_\perp \cdot \boldsymbol{k}_\perp f(k^2) = \pi \boldsymbol{a}_\perp \cdot \boldsymbol{b}_\perp \int dk_\perp^2 k_\perp^2 f(k^2). \tag{28}$$

If the constant vectors a and b are the Dirac gamma matrices we write for two four-vector products

$$\int d^2\boldsymbol{k}_\perp (\gamma)_1 \cdot k \ (\gamma)_2 \cdot k f(k^2) = \pi (\gamma)_1 \cdot (\gamma)_2 \int dk_\perp^2 k^2 f(k^2). \tag{29}$$

modulo $\gamma^\pm$ terms that we ignore at first. Here the sub-indices stand for the first or second matrix element in nucleon wave function components where the γ's occur.

Using

$$\int d^2\boldsymbol{k}_\perp k \cdot P \gamma \cdot k f(k^2) = \gamma \cdot P \ \pi \int dk_\perp^2 k^2 f(k^2), \tag{30}$$

the remaining spin invariants in Eq. 22 are now approximated by

$$\begin{aligned}
\int d^2\boldsymbol{k}_\perp k \cdot P \ \bar{u}_1' \gamma \cdot k \ C \bar{u}_2^T \ f(k^2) &= \bar{u}_1' \gamma \cdot P C \bar{u}_2^T \ \pi \int dk_\perp^2 k^2 f(k^2), \\
\int d^2\boldsymbol{k}_\perp k \cdot P \ \bar{u}_1' i\sigma_{\mu\nu} k^\nu C \bar{u}_2^T \ f(k^2) &= \bar{u}_1' i\sigma_{\mu\nu} P^\nu C \bar{u}_2^T \ \pi \int dk_\perp^2 k^2 f(k^2), \\
\int d^2\boldsymbol{k}_\perp \bar{u}_1' \gamma \cdot k C \bar{u}_2^T \ \bar{u}_3 \gamma_5 \gamma \cdot k u_N \ f(k^2) &= \bar{u}_1' \gamma_\mu C \bar{u}_2^T \ \bar{u}_3 \gamma_5 \gamma^\mu u_N \ \pi \int dk_\perp^2 k^2 f(k^2), \\
\int d^2\boldsymbol{k}_\perp \bar{u}_1' i\sigma_{\mu\nu} P^\mu k^\nu C \bar{u}_2^T \ \bar{u}_3 \gamma_5 \gamma \cdot k u_N \ f(k^2) &= \bar{u}_1' i\sigma_{\mu\nu} P^\mu C \bar{u}_2^T \ \bar{u}_3 \gamma_5 \gamma^\nu u_N \ \pi \int dk_\perp^2 k^2 f(k^2).
\end{aligned} \tag{31}$$

At this stage we have to specify $f(k^2)$ for each case in more detail. For $v_5(12)$ the kinematics $p_1' = p_1 - k$, $p_2' = p_2 + k$ mentioned above imply that E_{12} (see Eq. 14) and $R_0(q_3^2, Q_3^2)$ will depend on $\boldsymbol{k}_\perp$ via $\boldsymbol{p}_{1\perp}' \cdot \boldsymbol{k}_\perp$, $\boldsymbol{p}_{2\perp}' \cdot \boldsymbol{k}_\perp$, and $\boldsymbol{q}_{3\perp}' \cdot \boldsymbol{k}_\perp$ where the final momenta (with $'$) are held fixed. In fact, $(E_{12})^{-1} + (E_{21})^{-1}$, R_0 and M_0 depend on $q_3' \cdot k$, as required

by translation invariance. Note that E_{12} contains $-\boldsymbol{k}_\perp \cdot \boldsymbol{p}'_{2\perp}$ via E_2, while E_{21} contains $+\boldsymbol{k}_\perp \cdot \boldsymbol{p}'_{1\perp}$ via E_1, etc. which combine to $\boldsymbol{q}'_3 \cdot \boldsymbol{k}_\perp = -q'_3 \cdot k$.

Therefore, we may display the angular $\boldsymbol{k}_\perp$ dependence by expanding

$$R_0(q_3^2, Q_3^2) = (R_0)_0 + q'_3 \cdot k (R_0)_{q_3} + Q'_3 \cdot k (R_0)_{Q_3} + \cdots, \tag{32}$$

where $(R_0)_0$, $(R_0)_{q_3}$ etc. depend on the relative momentum variables $q_3'^2, Q_3'^2$ and on $k_\perp^2$ only without angular $k_\perp$ dependence. Similar expansions are valid for suitable combinations of energy denominators and for $M_0(q_3^2, Q_3^2)$.

Now the last four terms in Eq. 22 in conjunction with Eq. 31 after the angular integration may be written as

$$\int d^2\boldsymbol{k}_\perp k \cdot P \bar{u}'_1 \gamma \cdot k C \bar{u}_2'^T \ \bar{u}'_3 \gamma_5 u_N \, R_0 \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)$$
$$= \bar{u}'_1 \gamma \cdot P C \bar{u}_2'^T \ \bar{u}'_3 \gamma_5 u_N \, \pi \int dk_\perp^2 k^2 (R_0)_0 \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)_0 + \cdots, \tag{33}$$
$$\int d^2\boldsymbol{k}_\perp k \cdot P \bar{u}'_1 i\sigma_{\mu\nu} k^\nu C \bar{u}_2'^T \ \bar{u}'_3 \gamma_5 \gamma^\mu u_N \, R_0 \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)$$
$$= \bar{u}'_1 i\sigma_{\mu\nu} P^\nu C \bar{u}_2'^T \ \bar{u}'_3 \gamma_5 \gamma^\mu u_N \, \pi \int dk_\perp^2 k^2 (R_0)_0 \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)_0 + \cdots, \tag{34}$$
$$\int d^2\boldsymbol{k}_\perp \bar{u}'_1 \gamma \cdot k C \bar{u}_2'^T \ \bar{u}'_3 \gamma_5 \gamma \cdot k u_N R_0 \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)$$
$$= \bar{u}'_1 \gamma_\mu C \bar{u}_2'^T \ \bar{u}'_3 \gamma_5 \gamma^\mu u_N \, \pi \int dk_\perp^2 k^2 (R_0)_0 \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)_0 + \cdots \tag{35}$$
$$\int d^2\boldsymbol{k}_\perp \bar{u}'_1 i\sigma_{\mu\nu} P^\mu k^\nu C \bar{u}_2'^T \ \bar{u}'_3 \gamma_5 \gamma \cdot k u_N \, R_0 \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)$$
$$= \bar{u}'_1 i\sigma_{\mu\nu} P^\mu C \bar{u}_2'^T \ \bar{u}'_3 \gamma_5 \gamma^\nu u_N \, \pi \int dk_\perp^2 k^2 (R_0)_0 \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)_0 + \cdots. \tag{36}$$

Before we comment on Eqs. 33 to 36 let us continue the π^+ exchange started with Eq. 27. Integrating Eq. 27 yields

$$\mathcal{I}_2 \equiv \int d^2\boldsymbol{k}_\perp \left[v_5^+(23) \left(\frac{1}{E_{23}} + \frac{1}{E_{32}} \right) + v_5^+(13) \left(\frac{1}{E_{13}} + \frac{1}{E_{31}} \right) \right] R_0 \psi_N$$
$$= -\bar{u}'_2 \gamma_5 \gamma_\mu [P] \gamma_5 C \bar{u}_1'^T \ \bar{u}'_3 \gamma^\mu \gamma_5 u_N \, \pi \int dk_\perp^2 k^2 (R_0)_0 \left(\frac{1}{E_{23}} + \frac{1}{E_{32}} \right)_0$$
$$-\bar{u}'_1 \gamma_5 \gamma_\mu [P] \gamma_5 C \bar{u}_2'^T \ \bar{u}'_3 \gamma^\mu \gamma_5 u_N \, \pi \int dk_\perp^2 k^2 (R_0)_0 \left(\frac{1}{E_{13}} + \frac{1}{E_{31}} \right)_0$$
$$-\bar{u}'_2 \gamma_5 \gamma_\lambda [P] \gamma^\lambda C \bar{u}_3'^T \ \bar{u}'_1 u_N \, \pi \int dk_\perp^2 k^2 (R_0)_0 \left(\frac{1}{E_{23}} + \frac{1}{E_{32}} \right)_0$$

$$-\bar{u}_1'\gamma_5\gamma_\lambda[P]\gamma^\lambda C\bar{u}_3'^T\ \bar{u}_2'u_N\,\pi\int dk_\perp^2 k^2(R_0)_0\left(\frac{1}{E_{13}}+\frac{1}{E_{31}}\right)_0+\cdots. \tag{37}$$

In Eq. 37 the first two terms can be simplified and the last two Fierz rearranged to the canonical (12)3 order, so that Eq. 37 may be symbolically written as

$$\int d^2\boldsymbol{k}_\perp\mathcal{G}_0\left(v_5^+(23)+v_5^+(13)\right)(G_2+G_6)\sim\frac{3}{2}G_1\ r_-+\frac{3}{2}G_3\ r_+$$
$$+G_6\ R_-+\frac{1}{4}G_7\ r_++2G_8\ R_++\cdots, \tag{38}$$

where $r_\pm, R_\pm$ are radial integrals defined by

$$r_\pm=\pi\int dk_\perp^2 k^2(M_0R_0)_0\left[\left(\frac{1}{E_{23}}+\frac{1}{E_{32}}\right)_0\pm\left(\frac{1}{E_{13}}+\frac{1}{E_{31}}\right)_0\right], \tag{39}$$

$$R_\pm=\pi\int dk_\perp^2 k^2(R_0)_0\left[\left(\frac{1}{E_{23}}+\frac{1}{E_{32}}\right)_0\pm\left(\frac{1}{E_{13}}+\frac{1}{E_{31}}\right)_0\right]. \tag{40}$$

Clearly Eqs. 33 to 36 separate the last four terms of Eq. 22 into the nucleon spin invariants G_3, G_5, G_8 and radial integrals. The trem with v_5^+ now is seen to contain many other proton spin invariants other than G_2. The ellipses indicate new N*-type spin invariants generated from higher order terms in expansions like Eq. 32, which we address next.

When terms linear in $k\cdot q_3'$ and $k\cdot Q_3'$ are considered as exhibited in Eq. 32, then the relevant replacement of the identity Eq. 30 is

$$\int d^2\boldsymbol{k}_\perp k\cdot q_3'\gamma\cdot kf(k^2)=\gamma\cdot q_3'\ \pi\int dk_\perp^2 k^2 f(k^2), \tag{41}$$

etc. This entails replacing the two $\gamma\cdot k$ factors in expressions like Eq. 16 and 27 resulting from Eq. 13 by $\gamma\cdot q_3'$'s or $\gamma\cdot q_3'\ \gamma\cdot Q_3'$, etc. in the coupled radial equations of motion. The resulting spin invariants are characteristic of those of N*'s [15] with one or two quarks in higher orbitals. Thus, N* spin invariants in coupled equations for the nucleon originate from the entanglement of momentum wave functions (i. e. orbital angular momentum) with the 3-quark spin structure.

The first higher order terms contributing to Eq. 33, for example, are given by

$$q_3' \cdot Q_3' \bar{u}_1' \gamma \cdot P C \bar{u}_2'^T \ \bar{u}_3' \gamma_5 u_N \, \pi \int dk_\perp^2 k^4 (R_0)_{Q_3} \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)_{q_3}$$
$$+ Q_3' \cdot P \bar{u}_1' \gamma \cdot q_3' C \bar{u}_2'^T \ \bar{u}_3' \gamma_5 u_N \, \pi \int dk_\perp^2 k^4 (R_0)_{Q_3} \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)_{q_3}$$
$$+ q_3' \cdot P \bar{u}_1' \gamma \cdot Q_3' C \bar{u}_2'^T \ \bar{u}_3' \gamma_5 u_N \, \pi \int dk_\perp^2 k^4 (R_0)_{Q_3} \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)_{q_3} + \cdots, \tag{42}$$

where the second invariant vanishes when the Dirac equation is applied and the last invariant corresponds to a component of the negative parity N*(1535) configuration S_{11}. However, upon integrating there are higher order continuations for every term in Eq. 22. E. g. the second term continues with

$$\int d^2 \boldsymbol{k}_\perp k^2 R_0 \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right) = \cdots + q_3' \cdot Q_3' \ \pi \int dk_\perp^2 k^4 (R_0)_{Q_3} \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)_{q_3}$$
$$+ q_3'^2 \ \pi \int dk_\perp^2 k^4 (R_0)_{q_3} \left(\frac{1}{E_{12}} + \frac{1}{E_{21}} \right)_{q_3} + \cdots. \tag{43}$$

Clearly, these terms represent higher order contributions from the expansion of the momentum dependence of the radial wave function and energy denominators.

Next we turn to the gluon exchange as an example of a vector interaction. Again we start from the 1-2 symmetric $v_G(12)$ for simplicity,

$$v_G(12)\psi_N \equiv 4m_q^2 \, \bar{u}_1' \gamma^\mu u_1 \ \bar{u}_2' \gamma_\mu u_2 \left(\bar{u}_3[P] \gamma_5 C \bar{u}_2^T \ \bar{u}_1 u_N + \bar{u}_3[P] \gamma_5 C \bar{u}_1^T \ \bar{u}_2 u_N \right)$$
$$= -\bar{u}_3[P] \gamma \cdot k \gamma_\mu \gamma_5 C \bar{u}_2'^T \ \bar{u}_1' \gamma^\mu \gamma \cdot k u_N + 4 \bar{u}_3[P] \gamma_5 C \bar{u}_2^T \ \bar{u}_1' u_N \, p_1' \cdot p_2' + (1 \leftrightarrow 2)$$
$$+ 2\bar{u}_3[P] \gamma_5 C \bar{u}_2'^T \ \bar{u}_1' \gamma \cdot p_2' \gamma \cdot k u_N - 2\bar{u}_3[P] \gamma \cdot k \gamma \cdot p_1' \gamma_5 C \bar{u}_2'^T \ \bar{u}_1' u_N - (1 \leftrightarrow 2). \tag{44}$$

By Fierz rearrangement Eq. 44 leads to the following results with canonical (12)-spin matrix elements

$$v_G(12)\psi_N = -\bar{u}_1' \gamma_\lambda C \bar{u}_2'^T \ \bar{u}_3[P] \gamma \cdot k \gamma_5 \gamma^\lambda \gamma \cdot k u_N + 2 p_1' \cdot p_2' \Big[2M_0 \bar{u}_1' \gamma_\lambda C \bar{u}_2'^T \ \bar{u}_3 \gamma_5 \gamma^\lambda u_N$$
$$- 2\bar{u}_1' \gamma \cdot P C \bar{u}_2'^T \ \bar{u}_3 \gamma_5 u_N + \bar{u}_1' i \sigma_{\mu\nu} P^\nu C \bar{u}_2'^T \ \bar{u}_3 \gamma_5 \gamma^\mu u_N \Big]$$
$$+ \frac{1}{2} \bar{u}_1' C \bar{u}_2'^T \ \bar{u}_3[P] \Big[-\gamma \cdot k \gamma \cdot (p_1' + p_2') \gamma_5 + \gamma_5 \gamma \cdot (p_1' + p_2') \gamma \cdot k \Big] u_N$$
$$+ \frac{1}{2} \bar{u}_1' \gamma_\mu C \bar{u}_2'^T \ \bar{u}_3[P] \Big[-\gamma \cdot k \gamma \cdot (p_1' - p_2') \gamma_5 \gamma^\mu - \gamma_5 \gamma^\mu \gamma \cdot (p_1' - p_2') \gamma \cdot k \Big] u_N$$
$$+ \frac{1}{4} \bar{u}_1' \sigma_{\mu\nu} C \bar{u}_2'^T \ \bar{u}_3[P] \Big[-\gamma \cdot k \gamma \cdot (p_1' - p_2') \gamma_5 \sigma^{\mu\nu} - \gamma_5 \sigma^{\mu\nu} \gamma \cdot (p_1' - p_2') \gamma \cdot k \Big] u_N$$

$$+\frac{1}{2}\,\bar{u}_1'\gamma_5\gamma_\mu C\bar{u}_2'^T\;\bar{u}_3[P]\Big[\gamma\cdot k\gamma\cdot(p_1'+p_2')\gamma^\mu-\gamma^\mu\gamma\cdot(p_1'+p_2')\gamma\cdot k\Big]u_N$$
$$+\frac{1}{2}\,\bar{u}_1'\gamma_5 C\bar{u}_2'^T\;\bar{u}_3[P]\Big[-\gamma\cdot k\gamma\cdot(p_1'+p_2')+\gamma\cdot(p_1'-p_2')\gamma\cdot k\Big]u_N, \qquad (45)$$

which contain terms linear in k and the p_i'. As was the case for the pion exchange, for gluon exchange the explicit $\gamma\cdot p_i'$ factors lead to N*-type spin invariants here also. In the $d^2\boldsymbol{k}_\perp$-integration, a la Eq. 29, the linear term $\gamma\cdot k$ combines with $q_3'\cdot k$ from the R_0 expansion Eq. 32, when $\gamma\cdot(p_1'-p_2')$ is present as well in Eq. 45, or with $Q_3'\cdot k$ when $\gamma\cdot(p_1'+p_2')$ is present, etc.[3] Thus, Eq. 45 leads to the following spin matrix elements which can be readily manipulated into canonical form:

$$\bar{u}_1'\gamma_\lambda C\bar{u}_2'^T\;\bar{u}_3[P]\gamma_\mu\gamma_5\gamma^\lambda\gamma^\mu u_N,$$

for the first term, i. e. the vector invariant G_3 of Table I, while the third term gives G_5 and the fourth G_8. The second term in Eq. 45 also gives G_3. For the scalar (12)-term $\bar{u}_1'C\bar{u}_2'^T$ the associated second spin matrix elements reduce to

$$-\bar{u}_3[P]\gamma\cdot Q_3'\gamma\cdot(p_1'+p_2')\gamma_5 u_N+\bar{u}_3[P]\gamma_5\gamma\cdot(p_1'+p_2')\gamma\cdot Q_3'u_N=-4M_0^2(1-x_3')\bar{u}_3\gamma_5\gamma\cdot Q_3'u_N, \quad (46)$$

i.e. an N*-spin invariant. For the vector (12)-term $\bar{u}_1'\gamma_\mu C\bar{u}_2'^T$ they are

$$-\bar{u}_3'[P]\gamma\cdot q_3'\gamma\cdot(p_1'-p_2')\gamma_5\gamma^\mu u_N-\bar{u}_3'[P]\gamma_5\gamma^\mu\gamma\cdot(p_1'-p_2')\gamma\cdot q_3'u_N$$
$$=[-4P^\mu q_3'\cdot(p_1'-p_2')+4q_3'^\mu P\cdot(p_1'-p_2')]\bar{u}_3'\gamma_5 u_N$$
$$+4M_0q_3'\cdot(p_1'-p_2')\bar{u}_3'\gamma_5\gamma^\mu u_N-4M_0q_3'^\mu\bar{u}_3'\gamma_5\gamma\cdot(p_1'-p_2')u_N \qquad (47)$$

i. e. G_5, G_1, G_3 N-spin and an N*-spin invariant. Terms with $\gamma\cdot q_3'$ replaced by $\gamma\cdot Q_3'$ are similarly reduced to the canonical forms of Table 1. The tensor (12)-term of Eq. 46 generates G_7 and different N*-spin invariants, the axialvector (12)-term generates G_6 and

[3]Strictly speaking, a four-vector p must be replaced by $\tilde{p}=p-Pp\cdot P/P^2$ to maintain orthogonality. We omit the tilde for simplicity.

N*-spin invariants, while the pseudoscalar (12)-term generates G_2 and three different N*-spin invariants. In summary the factorization into radial integrals and spin invariants for the gluon exchange may be written symbolically as follows

$$
\begin{aligned}
\int d^2\boldsymbol{k}_\perp v_G(12)\psi_N = \pi\int dk_\perp^2 (R_0)_0\Big(\frac{1}{E_{12}}+\frac{1}{E_{21}}\Big)_0\Big[&4M_0(k^2+p_1'\cdot p_2')G_3 \\
&-4(k^2+p_1'\cdot p_2')G_5+2i\; p_1'\cdot p_2' G_7\Big] \\
+(R_0)_{Q_3}\Big(\frac{1}{E_{12}}+\frac{1}{E_{21}}\Big)_0&\Big[-2M_0^2(1-x_3')\bar u_1'\gamma\cdot PC\bar u_2^{'T}\;\bar u_3'\gamma_5\gamma\cdot Q_3' u_N\Big] \\
+(R_0)_{q_3}\Big(\frac{1}{E_{12}}+\frac{1}{E_{21}}\Big)_0&\Big[-2q_3'\cdot(p_1'-p_2')G_5+2\frac{(x_2'-x_1')m_q}{(1-x_3')}P\cdot(p_1'-p_2')G_1 \\
+2M_0q_3'\cdot(p_1'-p_2')G_3&-2M_0\frac{(x_2'-x_1')m_q}{1-x_3'}\bar u_1'C\bar u_2^{'T}\;\bar u_3'\gamma_5\gamma\cdot(p_1'-p_2')u_N\Big] \\
+(R_0)_{Q_3}\Big(\frac{1}{E_{12}}+\frac{1}{E_{21}}\Big)_0&\Big[-2M_0\bar u_1'\gamma_5C\bar u_2^{'T}\;\bar u_3'\gamma\cdot Q_3'\gamma\cdot(p_1'+p_2')u_N \\
+M_0Q_3'\cdot(p_1'+p_2')\bar u_1'\gamma_5C\bar u_2^{'T}\;&\bar u_3'u_N-2Q_3'\cdot P\bar u_1'\gamma_5C\bar u_2^{'T}\;\bar u_3'\gamma\cdot(p_1'+p_2')u_N \\
+2P\cdot(p_2'+p_2')\bar u_1'\gamma_5C\bar u_2^{'T}\;&\bar u_3'\gamma\cdot Q_3'u_N\Big]+\ldots
\end{aligned}
\tag{48}
$$

V. CONCLUSIONS

The light front form provides a framework to treat few-body systems in a relativistic covariant way. This approach is particularly suited when boosts are important, which is the case, e.g., for form factor calculations. So far nucleon light front wave functions have been treated in a restricted way in this context, namely using positive energy projected wave function components only that can be directly derived from the Pauli-Melosh basis. Besides this restriction, the spatial part of the wave function have been assumed symmetric and in the "ground state". In this paper we have addressed this issue and shown that residual interactions between the quarks lead to new components in the wave functions. These can be compared to configuration mixing in the nonrelativistic approach and have not been studied so far. They are expected to play an important role for the mass splittings, the $E2/M1$ ratio of the ΔN transition, the different high energy behavior of magnetic and electric form factors of the nucleon, and other important structure information of the baryons. For example,

since small Dirac components are included in the spin-flavor invariants, relativistic effects such as some pair currents are already included in the impulse approximation. (Compare Refs. [8,18] for the two-nucleon case.) We have investigated two currently used residual interactions, i.e. γ_5 coupling representing a typical Goldstone boson exchange and γ_μ coupling representing the gluon exchange and found many new spin-isospin structures not present in the simple nucleon wave function.

The *uds* basis used in this context suggests working in the Faddeev formalism for the three body system. We have evaluated a few of the integrals appearing in the respective equations utilizing light front time ordered perturbation theory. As a full solution of the problem still needs further work a next step in this direction is a one iteration approximation that has been proven quite useful in the two nucleon case and might be useful in the quark context also, because of the perturbative nature of the Goldstone boson exchange (that is based on chiral perturbation theory).

VI. ACKNOWLEDGEMENTS

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TABLES

TABLE I. Relativistic spin invariants for the nucleon. All invariants are symmetric with respect to exchange of 12. With respect to spin or isospin alone G_2, G_4, and G_6 are antisymmetric in the pair 12, the others are symmetric.

G_1 :	$1G$	$\otimes$	$\gamma_5 u_\lambda$
G_2 :	$\gamma_5 G$	$\otimes$	u_λ
G_3 :	$\gamma^\mu \boldsymbol{\tau} G$	$\otimes$	$\gamma_5 \gamma_\mu \boldsymbol{\tau} u_\lambda$
G_4 :	$\gamma^\mu \gamma_5 G$	$\otimes$	$\gamma_\mu u_\lambda$
G_5 :	$\gamma \cdot P \boldsymbol{\tau} G$	$\otimes$	$\gamma_5 \boldsymbol{\tau} u_\lambda$
G_6 :	$\gamma \cdot P \gamma_5 G$	$\otimes$	u_λ
G_7 :	$\sigma^{\mu\nu} \boldsymbol{\tau} G$	$\otimes$	$\gamma_5 \sigma_{\mu\nu} \boldsymbol{\tau} u_\lambda$
G_8 :	$i\sigma^{\mu\nu} P_\nu \boldsymbol{\tau} G$	$\otimes$	$\gamma_5 \gamma_\mu \boldsymbol{\tau} u_\lambda$

TABLE II. Fierz rearrangements for scalar (S), vector (V), tensor (T), axial vector (A), and pseudoscalar (P) operators (matrices). For definitions see Ref. [12].

	S	V	T	A	P
S	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{4}$
V	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
T	3	0	$-\frac{1}{2}$	0	3
A	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	1
P	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$

FIGURES

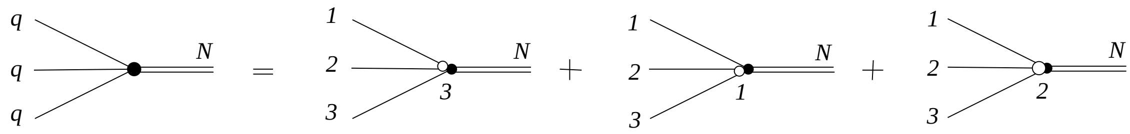

FIG. 1. Faddeev components of the wave function, see Eq. 8.

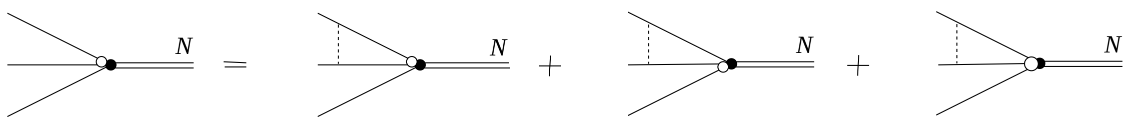

FIG. 2. Pictorial demonstration of Eq. 11 for $\Psi_N^{(3)}$.

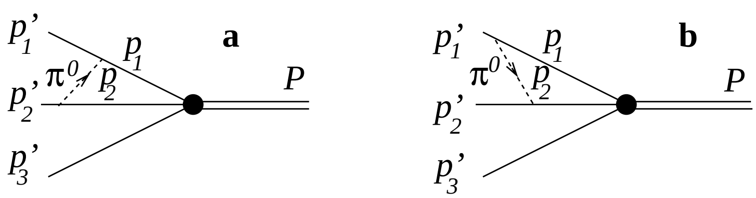

FIG. 3. Light cone time ordered neutral pion exchange between quarks 1 and 2.

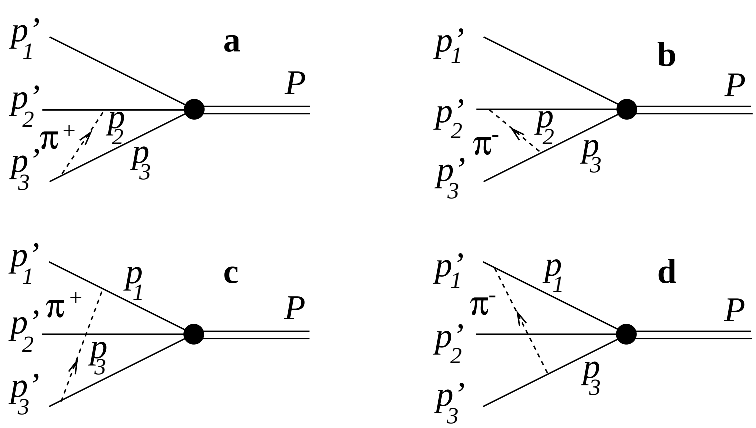

FIG. 4. Light cone time ordered charged pion exchange.

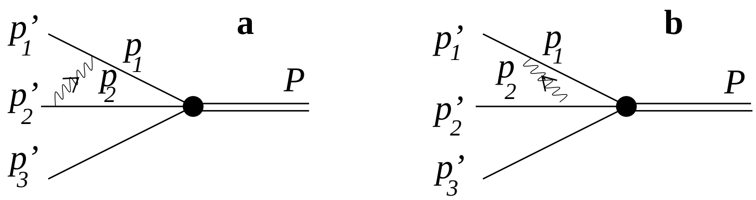

FIG. 5. Gluon exchange between quarks 1 and 2.

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